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FROZEN STORAGE OF POULTRY

V. EFFECTS OF SOME PROCESSING FACTORS ON QUALITY[1]

BY JESSE A. PEARCE[2] AND C. G. LAVERS[3]

Abstract

Examination of a number of phases of poultry processing showed that birds of optimum quality were obtained by: using a 90 sec. bleeding time; dipping in a semi-scald tank for 40 sec. at 128° F.; operating both shafts of the rough-plucker at the same speeds; operating the rough-plucker at conditions such that the rubber fingers did not exert a force greater than 11 lb. on the carcass; and carefully training operators of hand-roughers. Splitting the carcasses up the back, eviscerating, and packing flat reduced the space required for packing to less than 50% of that required by present methods, and still permitted reassembly of the bird into a presentable form for roasting. An impact dynamometer was useful in detecting unfrozen spots in the birds. Injecting various solutions into the carcass before freezing, and measuring the drip from the defrosted muscle, showed that maintaining the proper ratio of salts and water in the muscle reduced drip. Bleeding time had no effect on development of off-odors in stored poultry but evisceration before storage markedly reduced development of off-odor on the eviscerated surface and in the thigh meat. Coating eviscerated birds before freezing with carrageenin gel containing 6% sodium chloride delayed development of off-odor on eviscerated surfaces of defrosted carcasses.

Introduction

Earlier papers in this series described the effects of precooling, rates of freezing, and frozen storage on the quality of poultry. However, little published information is available with respect to effects of killing and plucking procedures on quality of frozen poultry. The present paper describes studies on effects of different intervals between killing and entry into the semi-scald tank on dressing out and keeping qualities; detection of unfrozen cores in carcasses; methods that reduce the space required for packing and storage; and preservation of the eviscerated surface after defrosting.

Methods

Many of the appraisals were subjective, and, since they dealt with factors affecting the grading, were made by inspectors of the Dominion Department of Agriculture according to Canadian regulations (2). Other subjective appraisals were made by taste panels recruited from the staff of the National Research Laboratories.

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[2] *Biochemist, Food Investigations.*
[3] *Formerly Chemical Engineer, Food Investigations. Present address: Research Department, Maple Leaf Milling Co., Toronto, Ont.*

The various quantitative methods are described in those sections in which they were used.

The procedure used in dressing poultry in many Canadian processing plants and used in this study is briefly as follows. The birds are killed, allowed to bleed for a short period (bleeding time) passed through a tank of violently agitated warm water (semi-scald tank), and then between the rotating shafts of an automatic rough-plucker where hollow rubber fingers beat off most of the feathers. The fingers, about 8½ in. long and 5/8 in. in diameter, are placed in rows on the shafts, which are about 5½ in. in diameter. The feathers remaining after rough-plucking are removed by a hand-rougher or by hand. The hand-rougher usually consists of a revolving drum, about 22 in. in diameter, with rubber fingers about 4 in. long on its surface. The shape of these fingers varies—some are circular and hollow, others are square and solid. The birds are held against these fingers by hand and the remainder of the coarse feathers are removed. After the birds leave the hand-rougher, pin-feathers are removed by hand.

Some plants have additional equipment for eviscerating birds, but in these studies, evisceration was done in the laboratory by hand.

Prefreezing Processing

Effect of Bleeding Time on Quality

In this phase of the study, the intervals allowed between killing and entry of the carcass into the semi-scald tank were 30 and 90 sec., since the use of longer or shorter times or of more than two time intervals caused too much

TABLE I

EFFECT OF 30-SEC. AND 90-SEC. INTERVALS BETWEEN STICKING AND SEMI-SCALDING ON PROCESSING DEFECTS IN POULTRY

Defect	Defective birds, %	
	30-sec.	90-sec.
Chicken		
Poor feather removal	12.8	1.5
Scalding	55.5*	45.0*
Rubbing	52.0	39.7
Skin bleeding	43.3	15.2
Fowl		
Poor feather removal	9.2*	8.2*
Scalding	31.7*	29.4*
Rubbing	33.6	18.2
Skin bleeding	7.3	1.2

**No significant difference between the values for the 30-sec. and 90-sec. intervals.*

inconvenience in the commercial plant. Unfortunately, the automatic control on the semi-scald tank was not operating on the day of this trial, and the temperature in the tank had to be controlled by hand. This resulted in temperature variation in the water from 128° to 132° F. The birds remained in the tank for 45 sec. One shaft of the rough-plucker was turning at 330 r.p.m. and the other at 290 r.p.m. The hand-rougher was operated at 360 r.p.m.

In all, about 800 birds were processed, about 400 at each bleeding time. The results (Table I) showed that birds classed as chicken suffered more defects than birds classed as fowl. For chicken, the short bleeding time reduced the efficacy of feather removal by the rough-plucker. On the other hand, feather removal was impossible if bleeding time was prolonged for five minutes. Rubbing, i.e., abrasion of the skin by the rubber fingers, was greater in birds bled for 30 sec. and is believed to be a direct reflection of the increased handling required to remove feathers on the hand-rougher. A 90 sec. bleeding time effectively reduced the incidence of skin bleeding. In general, these results show that a 90 sec. bleeding time is more desirable than a 30 sec. bleeding time; and for chicken in particular, a 90 sec. bleeding time effected a marked reduction in the number of feathers that must be removed by the hand-rougher.

TABLE II

EFFECT OF TEMPERATURE AND TIME IN SEMI-SCALD TANK ON FORCE REQUIRED TO REMOVE FEATHERS FROM POULTRY

(10 to 40 birds processed at each condition)

Temp., ° F.	Time, sec.	Force, oz.		
		Plant A	Plant B	
		Chicken	Fowl	Chicken
125	30	15	22	24
	40	13		15
	50			15
128	20	12		
	30	14	20	23
	40	15		11
130	20			9
	30			11
132	15	13		10
	20	16		
	30	8		
	40	7		
134	15	15		
	20	8		
136	15	5		

Effect of Time and Temperature in the Semi-scald Tank

The time that birds are in the semi-scald tank and the temperature of the water affects not only the quality of the dressed birds but also the effectiveness of the plucking operations.

Table II shows the effect of conditions in the semi-scald tank on the average force required to remove small feathers from the wings and tail (feathers clamped to spring scale). Work was done at Plant A early in September, when birds were still moulting. Relatively little force was required to remove the feathers, and excessive times and temperatures were necessary to effect any reduction in force required. Work was done in Plant B in late November, and, at the lower temperatures and shorter dipping times, the force required to remove the feathers was generally much higher than for September birds. The effect of time of year on the ease of feather removal requires more study. However, for the two periods studied here, approximately uniform resistance might be obtained by the use of appropriate dipping times and temperatures. In general, these results show that the force required to remove feathers from the birds is reduced by increasing the water temperature in the semi-scald tank and by increasing the time of dipping.

Table III shows that the effect of semi-scald tank operation on damage (combined scalding and rubbing) to the carcass is complicated by plucking operations. In Plant A, the birds were examined after passage through a rough-plucker with shafts rotating at 380 r.p.m. and a hand-rougher operating at 130 r.p.m. In Plant B, the birds were examined after passage through a rough-plucker only, with shafts turning at 360 r.p.m. Birds processed in Plant A at the intermediate temperatures showed least damage at the longest dipping times, while birds processed in Plant B showed greatest damage at the longest dipping times. This discrepancy was apparently due to the use of the hand-rougher in Plant A. At the longer dipping times, feathers were more easily removed by the hand-rougher, which reduced the damage from rubbing. From these results and the results in Table II, dipping for 40 sec. in a tank containing water at 128° F. seemed most desirable, if the operation of the two types of pluckers was satisfactory.

Effect of Plucker Operation on Quality

This study was made in only one plant. To eliminate damage from scalding, the water in the semi-scald tank was at 125° F. and the dipping time 30 sec. This temperature and time combination is lower than that recommended in the previous section and feather removal in the rough-plucker was less complete.

Table IV shows that the amount by which the rubber fingers overlap, from a gap of 2½ in. to an overlap of 3½ in., had little effect on rough-plucker operation but that the two shafts had to be operated at the same speed if excessive rubbing was to be avoided. If the shafts were operating at the same speed, the least rubbing damage occurred when the rubber fingers did

TABLE III

EFFECT OF TEMPERATURE AND TIME IN SEMI-SCALD TANK ON FEATHER REMOVAL AND DAMAGE

(10 to 40 birds examined at each condition)

Temp., °F.	Time, sec.	Plant A	Plant B	
		Birds damaged, %	Feather removal	Birds damaged, %
125	30	10	Fair	10
	40	50	Good	58
128	20			
	30	60	Good	50
	40	25	Good	80
130	20		Good	66
	30		Good	90
132	15	100	Fair to Good	81
	20	80		
	30	88		
	40	66		
134	15	55		
	20	90		
136	15	100		

TABLE IV

FACTORS IN ROUGH-PLUCKER OPERATION AFFECTING FORCE ON THE BIRD, FEATHER REMOVAL, AND DAMAGE

(10 to 40 birds examined at each operating condition)

Rough plucker			Maximum downward force on birds, lb.	Feather removal	Damaged birds, %
Speed, r.p.m.	Overlap of fingers, in.				
	Entrance	Exit			
Shafts at same speed					
270	2½	0	7.0	Poor	0
	3½	1½	8.0	Poor	0
360	0	0	8.8	Fair	20
	1	1	10.6	Fair	10
440	2½	0	13.6	Good	85
Shafts at different speeds					
330, 340	1	0	9.8	Fair	80
	3½	1½	7.4	Fair	77
380, 410	0	−2½	11.2	Good	75
	2½	0	13.3	Good	77

not exert a maximum force (measured by a spring scale) of more than about 11 lb. on the bird's carcass.

Table V shows that use of the hand-rougher (335 r.p.m.) under semi-scalding conditions approximating those described above doubled the damage to both chicken and fowl. Since little mechanical change can be made in this hand-rougher, the operators must be given special training in its use to avoid excessive damage at this stage of the operation.

TABLE V

EFFECT OF HAND PLUCKER ON DAMAGE TO BIRDS
(30 birds of each type examined at each operating condition)

Equipment used	Birds damaged, %	
	Chicken	Fowl
Rough plucker only	36	42
Rough and hand plucker	62	83

Packing Poultry for Freezing

Current methods of packing poultry waste space, so it was of interest to examine other methods. For this purpose, New York dressed birds weighing 60 lb. per doz. were used, which, by current packing methods, require a box with inside dimensions of 24 by 14½ by 7½ in., i.e., approximately 2600 cu. in. (Fig. 1). Eviscerating the birds and removing the heads and feet reduced the size of the box required to 24 by 14½ by 5½ in., i.e., to approximately 1900 cu. in.

Further comparisons were made using eviscerated birds. Various methods, based on cutting the bird through the center of the ventral cavity or on cutting the bird completely in half, were tried and discarded, since the bird could not be reassembled to make presentable roast chicken and since the breast muscles were exposed with the added danger of spoilage in the most meaty portion of the bird. Splitting birds up the back reduced the time required for evisceration and permitted easy reassembly for roasting, but packing with the back halves of two birds in the body cavity of the third saved little space.

Cutting the birds up the back and flattening by cracking the ribs provided packing in a minimum of space and still permitted reassembly into a presentable form for roasting. Two methods of packing were selected from the large number possible. In the first, the flattened birds were packed on edge with the neck or thighs alternately toward the bottom of the box. This pack required a box with internal dimensions of 21 by 8 by 9½ in., i.e., approximately 1600 cu. in. In the second (Fig. 2), the birds were packed in three columns with the breast surface of one bird in the eviscerated cavity

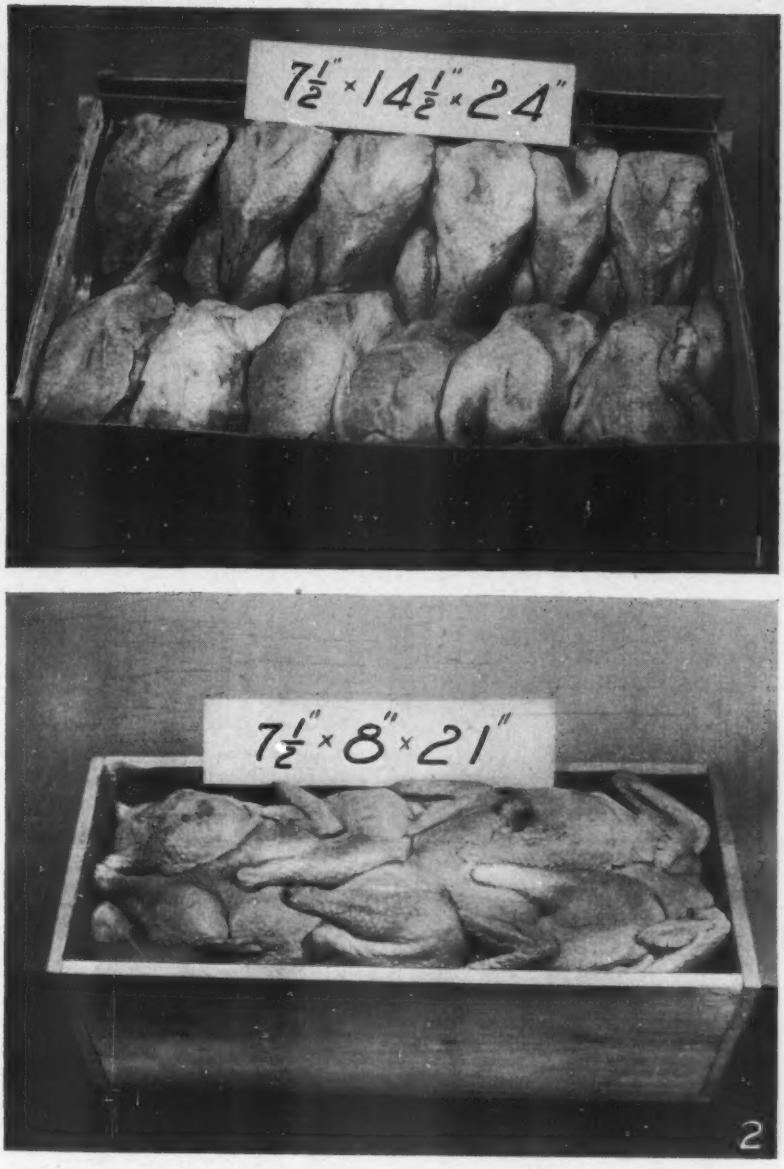

FIG. 1. *Current method of packing poultry: weight of birds, 60 lb. per doz.*
FIG. 2. *A suggested method of packing poultry: weight of birds, 60 lb. per doz.*

of another. Necks and legs in each row were alternated to reduce the space required. This pack fitted into a box with internal dimensions of 21 by 8 by 7½ in., i.e., approximately 1250 cu. in.

Both of these methods effect considerable saving in space and in packing material but have some disadvantages. In the common method of packing, the frozen birds can be separated fairly readily for sale, but in both the suggested methods the birds freeze into a solid block and cannot be separated readily unless completely defrosted, or unless two sheets of a light, dry-waxed paper are placed between the birds. The first of the suggested methods presents an unsightly appearance because of the protruding necks and joints; the second suffers from an objection to piling birds on top of each other. Neither method permits easy grading. However, these disadvantages are offset by the advantages; ease of evisceration, saving in packing material, and saving in space. In addition, the reduction of exposed surface should help reduce the freezer-burn that occurs during extended storage periods.

Freezing Poultry

Measuring Freezing

Several studies of factors affecting freezing rates in poultry have been made (5; 6; 7; 9, p. 438) and, in these, freezing time was defined as the time required for the temperature to be reduced by a definite amount. Development of a dynamometer for testing hardness and depth of freezing in frozen foods (8) permitted a comparison between these factors and freezing time as assessed by conventional temperature measurements.

In this phase of the study, three- and five-pound birds at about 31° F. were hung at intervals of 1 ft. in rooms operating at − 28°, −10°, 0°, and 20° F., with gentle air motion. Thermocouples were placed in the centers of the birds and the freezing time was that required to attain a temperature of 20° F. in the center of the bird. If poultry flesh is similar to the flesh of other animal products (6) about 10% of the water would still be unfrozen at this temperature. Freezing times by this method were determined with a standard deviation of ± 0.6 hr. and were, therefore, considered satisfactorily reproducible.

Dynamometer measurements were made at each sampling time in three parts of the birds' carcass: in the breast area, between the lower ribs, and in the abdomen. There was little difficulty in allowing for passage of the dynamometer through the bones; but air cavities in the viscera were a problem, since they offered no resistance to the dynamometer. The best point for measurement appeared to be between the lower ribs, but the results presented here are the average for all three sets of readings.

The temperature measurements (Fig. 3) showed that increasing the temperature of the freezing chamber from − 28° to 0° F. increased the freezing time from 5 to 12 hr; at 20° F. the freezing time was about 48 hours.

The times in which the birds were frozen to any fixed depth (assessed by the dynamometer) paralleled the freezing time; and the hardness of the frozen portion, as measured by resistance to dynamometer penetration, increased uniformly as freezing progressed at any given temperature. The marked variation in hardness of birds frozen at different temperatures had no effect

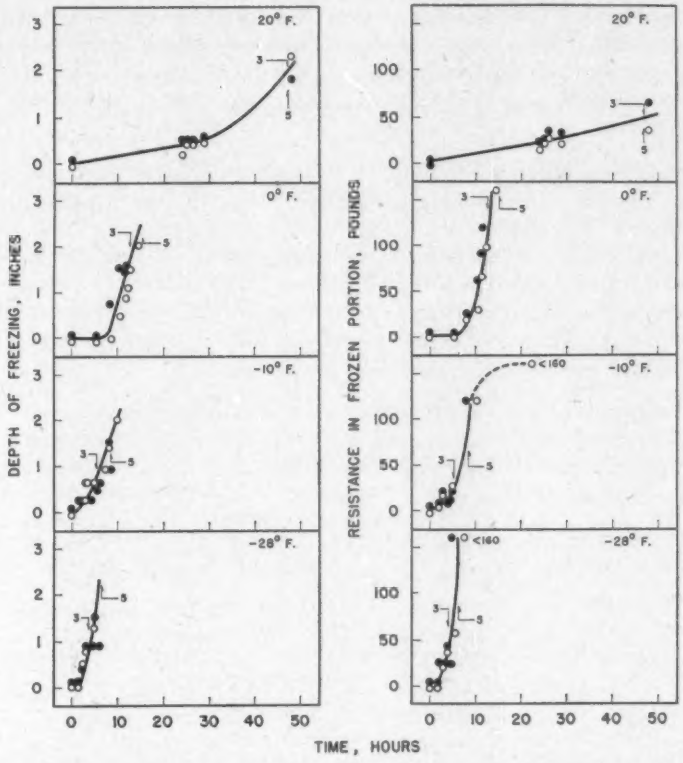

FIG. 3. *The freezing of 3-(o) and 5-(•) lb. chicken carcasses, as assessed by an impact dynamometer, and as related to the time for the center of the carcass to reach 20° F. determined with thermocouples: arrow marked 3, point at which 3-lb. birds reached this temperature; arrow marked 5, point at which 5-lb. birds reached this temperature.*

on the detection of unfrozen portions, which have no resistance to the dynamometer. It was concluded that, for poultry, the usefulness of the dynamometer will probably be confined to the detection of unfrozen spots in the carcasses.

Reducing Drip in Frozen Poultry

Studies elsewhere (7) have shown that open canals or vacuoles may be retained in muscle fibers by fast freezing. This retention of vacuoles was believed to be an indication of intrafibrillar freezing. Vacuoles may be present in the muscle fibers of live or freshly killed birds, although they may

not be detected by present histological techniques. Their presence in the muscles of rapidly frozen birds (7), and the reduced drip in defrosted birds that have been frozen rapidly (5), may be an indication that rapid freezing is necessary to preserve the vacuoles and, thereby, to reduce drip from the muscles after defrosting by retaining the fluid in vacuoles. If this is so, fluid might be retained by other procedures, such as the injection of solutions that would reduce pH changes, or alter the diffusion of ions that normally occurs in dead muscle tissue. This problem was examined by introducing various solutions into the birds' muscles, allowing the birds to stand for about an hour before freezing in about five hours, and by measuring drip after defrosting.

The solutions used were: water, 10^{-4} *N* ammonium hydroxide, phosphate buffer at pH 7.2 (3, p. 200), and Tyrode's modification of Ringer's solution (1, p. 257). The number of aliquots in any part of a carcass was roughly proportional to the amount of muscle believed to be in that portion of the bird and to a limiting amount of 30 ml. per lb. of muscle; i.e., its moisture content would be increased from approximately 70% to 85%. Each solution was used on groups of four birds; in addition four birds were frozen without injection, and four unfrozen birds were examined.

Drip was measured as the per cent decrease in weight by a technique similar to that described elsewhere (5) except that the right and left chest and thigh muscles were dissected from the frozen bird and drip measured without mincing.

The data were subjected to statistical analysis and the results are given in Table VI. Chest muscles lost a significantly greater quantity of fluid

TABLE VI

EFFECT OF ADDING 30 ML. OF VARIOUS SOLUTIONS PER LB. OF BIRD, BEFORE FREEZING, ON THE AMOUNT OF DRIP IN DEFROSTED POULTRY MUSCLES

(Four birds given each treatment)

Treatment	Amount of drip, %		
	Chest	Thighs	Average
Unfrozen None	2.5	1.4	2.0
Frozen None	3.4	2.0	2.7
Water	5.4	4.0	4.7
Ammonium hydroxide	5.0	4.6	4.8
Phosphate buffer	4.6	4.2	4.4
Tyrode's solution	4.1	3.6	3.8
Necessary difference, 5% level			0.8
Average	4.2	3.3	

than thigh muscles, but, although frozen untreated muscle exuded more fluid than unfrozen muscle, the difference was not statistically significant. The injection of water into muscle caused a marked increase in drip, but a large portion of the added water was retained by the muscle. The use of Tyrode's solution gave a product with significantly less drip than that containing added water, indicating that retaining the proper ratio of salts and water in muscle may help to retain its structure during freezing. Evidence of vacuoles was found in frozen sections of only the muscles injected with Tyrode's solution.

Retaining Quality After Freezing

Frozen Storage

Poultry in chill or frozen storage develops an undesirable odor in the flesh of the thigh. It has often been suggested by processors and others that this odor was attributable to the short interval that elaspes between killing the bird and introducing it into the semi-scald tank. It was believed that the bird entered the water with its heart still beating, that the dirty semi-scald water containing bacteria was pumped through the veins and arteries, and that, during storage, bacteria retained in the thigh caused the off-odor. On the other hand, the possibility exists that off-odors come from the viscera, as only a thin membrane separates the viscera from the meat in the thigh.

To examine this question, Grade A fowl processed in an earlier phase of this study were stored at 0° and 30° F. and examined at the intervals shown in Fig. 4. Forty birds had been subjected to the 30-sec. bleeding time and 40 birds to the 90-sec. bleeding time. Each group of birds was divided into four lots: two lots from each group were eviscerated; one lot of eviscerated and one lot of noneviscerated birds from each group was stored at each temperature. At each sampling time random pairs of birds were drawn from each lot and group, and the bacterial count in the thigh muscles was determined. All the birds were submitted to a taste panel of 24 persons who smelled the eviscerated surface and the thigh meat and scored it according to a scale from 0, no off-odor, to 5, gross off-odor.

The viable bacterial count varied from 10^2 to 10^3 per gm. of thigh meat and did not change during storage at either temperature.

The taste panel data were subjected to statistical analysis and the significant results are given in Fig. 4. They showed that bleeding time had no effect on the development of off-odor either on the eviscerated surface or in the thigh meat, but that off-odors increased rapidly in the viscera and were transmitted to the thigh meat. Therefore, eviscerating seems to be a desirable step in processing frozen poultry.

Storing Defrosted Birds

The eviscerated surface of poultry is usually believed to deteriorate rapidly unless the carcass is frozen, and this deterioration is reputed to be more rapid in defrosted birds than in birds that have never been frozen. Therefore,

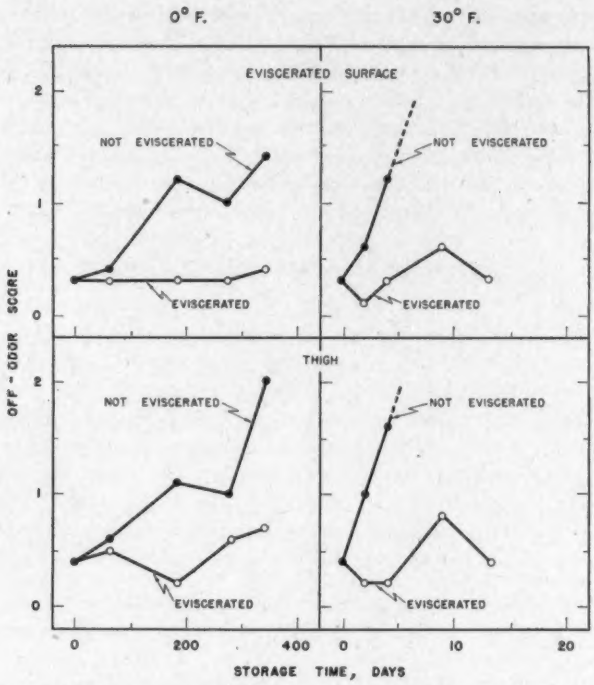

FIG. 4. *Change in off-odor on the eviscerated surface and in the thigh of eviscerated and non-eviscerated fowl stored at 0° and 30° F.*

it was of interest to determine the storage life of defrosted birds held at temperatures of 40°, 60°, and 80° F. and to examine possible methods of extending storage life at these temperatures.

For this study, birds weighing 60 lb. per dozen were eviscerated and treated before freezing as shown in Table VII. Wrapping was done by heat-sealing

TABLE VII

STORAGE LIFE, HR., OF DEFROSTED EVISCERATED POULTRY TREATED IN VARIOUS WAYS AND HELD AT DIFFERENT TEMPERATURES

Treatment	Temperatures, °F.		
	40	60	80
None	120	31	12
Partial cook	120	16	8
"Cellophane" wrap	48	25	12
Sodium benzoate dust	120	31	12
Butyl ester of *p*-hydroxybenzoic acid dust	120	31	12
Calcium proprionate dust	120	48	34
Carrageenin gel coat	168		
Carrageenin gel coat (4% sodium chloride)	168	72	
Carrageenin gel coat (6% sodium chloride)	192	83	32

the birds in 450 M.S.Y.T. "Cellophane". Carrageenin gel, an extract of Irish moss seaweed, was applied by dipping the birds into solutions, at 212° F., consisting of 4% of a 4 to 1 mixture of carrageenin and potassium chloride (4) to which 0, 4, and 6% of sodium chloride was added. The eviscerated birds were partially cooked by placing them in a chamber which was brought to 212° F. in 15 min. and held at that temperature for 30 min. The dry chemicals were dusted on the eviscerated surface only. Off-odors on the eviscerated surface were scored on a scale from 0 (good) to 4 (very badly off) and a score of 2 was considered the limit at which the birds would be suitable for table use. The numbers of viable bacteria on the eviscerated surface were determined at intervals, but the counts were too irregular to be of value.

Table VII shows the time in hours required for the birds to reach an off-odor score of two. Unwrapped, untreated birds kept for 120, 31, and 12 hr. at 40°, 60°, and 80° F. respectively. Wrapping in Cellophane or partial cooking caused more rapid deterioration than holding untreated birds in the open. Dusting with dry chemicals had no effect. Coating the birds with carrageenin gel containing 6% sodium chloride approximately doubled the storage life. The gel is easily removed, and in this study, as in another study of the use of salt for preserving quality in unfrozen birds (10), the salt did not affect the palatability of the flesh.

Acknowledgments

The authors wish to express their thanks to the many persons who assisted with various phases of this study, and special thanks to Mr. E. D. Bonnyman and his inspectors in the Special Products Board of Marketing Service, Department of Agriculture (Canada). The managers and staffs of the Ottawa and Smith Falls, Ontario, plants of Canada Packer's Limited, and the manager and staff of Le Co-operative des Produits Avicoles at Marieville, Que., also gave full co-operation throughout the work.

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A SEMIQUANTITATIVE METHOD FOR THE DETECTION OF ACETIC ANHYDRIDE IN ACETIC ACID[1]

By G. Benson[2] and R. M. Kitchen[3]

Abstract

When acetic acid is handled in aluminum, it is important, for reasons of corrosion resistance, that the acid be not completely anhydrous, a condition that results if traces of acetic anhydride are present. A quick colorimetric test depending on the reaction between acetic anhydride with aniline has been developed by which it is possible to detect 0.05% acetic anhydride in acetic acid.

In the manufacture, shipment, and use of glacial acetic acid considerable use is made of aluminum vessels and pipe lines, the corrosion resistance of which is very good when, as is normal, the acetic acid contains traces of water. In perfectly anhydrous acetic acid, however, the rate of corrosion is quite rapid (5, 6) and care must be taken in industrial practice to ensure that anhydrous acetic acid does not come into prolonged contact with aluminum. The only circumstances under which anhydrous conditions are likely to be produced and maintained is by the addition of acetic anhydride to acetic acid. The rate of corrosion of aluminum by acetic anhydride is greater than that by glacial acid (*c.* 99.7% acid) but is much smaller than the rate by anhydrous acid. From the corrosion standpoint therefore, it is the presence of small amounts of acetic anhydride in acetic acid that is particularly undesirable.

The curve shows the rate of corrosion of aluminum in the boiling liquid plotted against acidity expressed as acetic acid, 100% acetic anhydride being therefore represented by 117.6% acidity.

These results were obtained by boiling weighed samples of sheet aluminum totally immersed in mixtures of the acid with anhydride or with water in a flask fitted with a reflux condenser attached to a drying tube, and reweighing after intervals varying from one to several days. The approximately 100% acid was obtained by adding 1% acetic anhydride to 99.9% acid and in this solution the metal sample completely disintegrated in a few days.

Acetic acid is usually estimated by freezing point or by direct titration. The former will, of course, not distinguish between water and acetic anhydride as an impurity. Titration will permit of this distinction but very careful analytical technique is required, since the titration figure obtained in the presence of 0.1% acetic anhydride will differ from that for 0% anhydride by less than 0.02%.

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[2] *Director of Plant Research.*

[3] *Assistant Works Chemist.*

Acetic anhydride may be determined by direct titration or by the difference between total acidity and acidity after reaction with aniline. The second method also requires many precautions and has been found to be quite inadequate for the rapid estimation under plant conditions of small percentages of acetic anhydride.

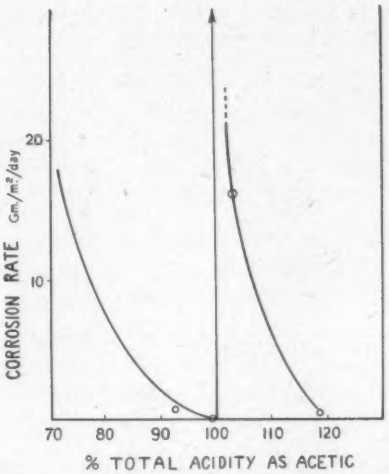

FIG. 1. *The corrosion of aluminum by acetic acid—water and acetic acid—acetic anhydride.*

For the measurement of low concentrations of acetic anhydride methods have been described in the literature involving reaction with dichloraniline followed by extraction with chloroform (3), or by determination of excess dichloraniline with bromide–bromate solution (2). Both are accurate but take longer than is desirable. A colorimetric test depending on the Liebermann–Burchard reaction for cholesterol (1) is not sufficiently sensitive.

Strength of acid, %	Corrosion rate, gm./m.²/day
50	50
64	12.3
92	0.8
99.7	0.03
100	287
103.5	16.5
116.5	1.5
118	0.7

The following method, depending on the reaction of acetic anhydride with aniline followed by a colorimetric test for free aniline by its reaction with furfural (4), was developed. In the hands of relatively unskilled analysts it will detect 0.5% acetic anhydride in acetic acid and with practice the limit

of detection can be reduced to one-tenth this amount. The color reaction of aniline with furfural was found to be more sensitive in this application than other more common tests.

Experimental

Two reagent solutions are required, one of 5% aniline in pyridine, and one of 1% redistilled furfuraldehyde in acetic acid.

If less than 0.5% anhydride is present in the acid, 1 ml. of the aniline solution is mixed with a 10 ml. sample and allowed to stand one minute. Then 1 ml. of the furfural solution is added. If no anhydride or less than 0.5%, is present in the acid a cherry-red color develops. If 0.5% or more of anhydride is present, a faint straw color will develop.

A test may be made for as little as 0.05% anhydride by using only 0.1 ml. of the aniline solution for a 10 ml. sample. In this case, of course, the color developed by a negative sample is much less intense.

The acetic anhydride in a sample may be estimated to within ±0.05% by using sufficient aniline to develop a color and choosing the best match to this color with the color from a series of standards, which should be freshly prepared, containing 0.0, 0.1, 0.2%, etc. anhydride.

However, we have generally found it sufficient to use the test as a limit gauge at a selected anhydride concentration such as 0.2%, using such quantities of aniline that no color will be developed if the anhydride be above this limit.

The following substances have been found not to interfere with the test when present in concentrations likely to be encountered in dealing with commercial grades of acetic acid; ethylidene diacetate, copper acetate, acetaldehyde (up to 1%) and water (up to 10%).

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THE MODIFIED LAMBERT CONFORMAL PROJECTION FOR POLAR AREAS[1]

By C. H. Ney[2]

Abstract

The search for a map suitable for aerial navigation over polar regions has led to the development of the Modified Lambert Conformal Projection for Polar Areas. In this paper the author points out the properties required in a map for polar navigation, and shows to what extent this projection satisfies these requirements.

Introduction

During the last war, the military importance of the Canadian Arctic was brought sharply into focus when it was found that a German meteorological station had been set up in Greenland to aid the Nazi Air Force in its attacks on the British Isles and to obtain information for possible attacks on the North American Continent. About the same time, the Allied Nations undertook the development of northern aerial routes linking Alaska and Northern Canada with Europe and Asia. As a result, it soon became apparent that the supply of maps of Arctic Canada suitable for aerial navigation was very limited and for the most part, unsatisfactory.

In 1945 and 1946, as a member of the Sub-committee on Navigation, Associate Committee on Aeronautical Research, National Research Council, the author undertook the development of a suitable polar projection for aerial navigation in Arctic Canada.

Most of the unsatisfactory features of long-range polar navigational maps had been well known for some time. Postwar experimental flights by the United States Army Air Force and the Royal Canadian Air Force from Continental Canada northward over the Arctic Archipelago have substantiated most of the known criticisms, which may be listed as follows:—

(*a*) Unsatisfactory basic map projections,
  (1) Where the meridians are delineated as curved lines, azimuths cannot readily be scaled from the map.
  (2) Lack of conformality.
  (3) Large distortion in scale at the outer edge of the map.

(*b*) Too small a scale.

(*c*) Congestion of meridians close to the Pole.

(*d*) Inadequate numbering of parallels of latitude and meridians. When the map is folded on the navigator's table, the numbering is often hidden from view.

(*e*) Congestion of isogonic lines near the geographic and magnetic poles.

(*f*) Inaccurate magnetic data.

(*g*) Inaccurate plotting of land masses.

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[2] *Chief.*

Polar projections such as the stereographic, the equidistant, and the gnomonic give fairly satisfactory representations of terrestrial regions close to the Pole; but where the area to be mapped is extended down to latitude 65°, the selection of a suitable projection becomes a more difficult matter. For the investigator of polar mapping, there is a projection—the Lambert Conformal with two standard parallels—which merits consideration. This projection was first developed about 1770 by Johann Heinrich Lambert, a German mathematician, and was used extensively by the French in the First World War. In studying the mapping of polar areas, this projection might easily be passed over with scant consideration. However, a careful study of the subject leads to the conclusion that Lambert's projection may be so modified and adapted to high latitudes that it outclasses most of the other possible choices. Most of the standard projections are developed on a tangent plane or cylinder. The fact that the Lambert projection is developed on a secant cone suggests at once a considerable reduction in scale errors over the map as a whole. The conformal characteristics of the projection lend assurance that azimuths of great circle courses will be accurately shown.

On Lambert's projection, the scale errors around any point are the same in all directions. At each standard parallel, there is no error in scale. Elsewhere on the map, the magnitudes and signs of the scale errors vary with the latitude and depend to a large extent upon the spacing of the standard parallels. The scale errors are positive in sign for the areas between the southerly standard parallel and the outer edge of the map and between the northerly standard parallel and the Pole, and negative for the area between the two standard parallels. By taking advantage of this feature, a high degree of compensation may be attained in the distribution of scale errors over the map as a whole.

Modification of the Lambert Projection

In the true Lambert Conformal projection, $\theta = L.\Delta\lambda$, where θ is the mapping angle, $\Delta\lambda$ the corresponding true difference of longitude, and where the numerical value of L depends upon selected latitudes for the two standard parallels. As the value of L is always less than unity, the projection could not be used for mapping 360° in longitude at the Pole without leaving a wedge-shaped, blank area extending from the Pole to the outer edge of the map. Obviously this would be a very unsatisfactory type of map for aerial navigation.

By shifting the northerly standard parallel closer and closer to the Pole the value of L can be made to approach unity. At the same time, the critical position of the southerly standard parallel which will minimize and compensate the scale errors over the map as a whole can be found.

In order to place the standard parallels at the most advantageous positions to satisfy the requirements enumerated, a number of possible arrangements were tried out. A sufficient number of scale errors were computed for each

choice to be of help in making subsequent selections. After four or five trials, the following latitudes were adopted:

Northern Standard Parallel	89°	59′	58″
Southern Standard Parallel	71°	00′	00″

It was not possible to place the northerly standard parallel exactly at the Pole, as some of the formulae used in the computation of the map elements would then become indeterminate.

Resulting from this choice, the value of L was found to be 0.99737541—a close approximation to unity.

In the computation of the radii of the parallels of latitude and the scale errors, the value 0.99737541 was used throughout, in strict accordance with the mathematics of the true Lambert projection. In establishing the positions of the meridians, a slight deviation from the orthodox projection has been made. At a very slight sacrifice of conformality, the meridians have been established on the assumption that L equals unity. This means that the mapped value of the difference of longitude between two meridians is arbitrarily made equal to the true difference of longitude on the earth's surface. This modification, while altering the true Lambert projection very slightly with regard to conformality and scale, permits of 360° mapping around the Pole without leaving an undesirable gap extending from the Pole to the outer edge of the map.

The actual construction of the graticule is very simple. With the Pole as the center, concentric circles are drawn having radii equal to the calculated values. The meridians are then drawn as straight lines radiating from the Pole with the mapped differences of longitude equal to the corresponding values on the earth.

Scale of the Map

In conformity with the suggestions of a number of navigation officers of the R.C.A.F. and the U.S.A.A.F., a scale of 30 nautical miles to the inch has been adopted. A preliminary projection drawn on a scale of 60 nautical miles to the inch was tried out and found to be inadequate.

The Effect of Modification on the Scaled Distances

Fig. 1 shows the relation between the mapping angles θ and θ' on the true and modified projections, respectively.

The difference between the corresponding lengths of the great circle courses AB and AB' represents the distortion in scale due to modification. We then have

$$AB' - AB = 2R\cos\left(90° - \frac{\Delta\lambda}{2}\right) - 2R\cos\left(90° - \frac{L.\Delta\lambda}{2}\right)$$

$$= 2R\left(\sin\frac{\Delta\lambda}{2} - \sin\frac{L.\Delta\lambda}{2}\right).$$

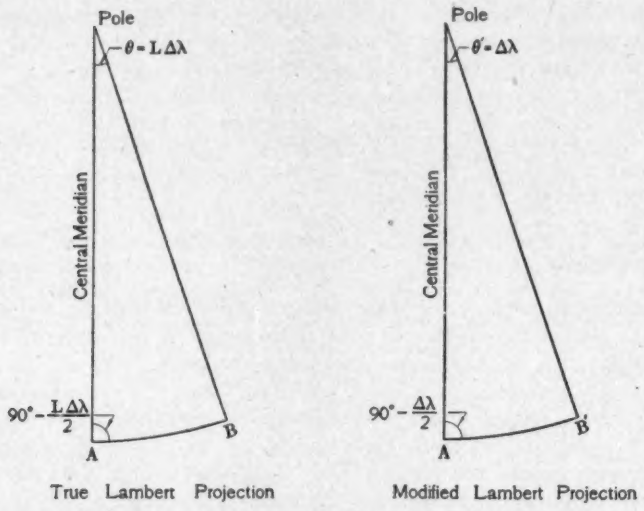

FIG. 1. *Mapping angle on the true and Modified Lambert projection.*

The ratio of distortion, due to modification, to course length, $\frac{AB' - AB}{AB}$, will be a maximum for a short east–west course. For such a course, $\Delta\lambda$ and $L.\Delta\lambda$ are so small that their values in radian measure may be substituted for their sines, so that

$$\frac{AB' - AB}{AB'} = 2R\frac{\left(\frac{\Delta\lambda}{2} - \frac{L.\Delta\lambda}{2}\right)}{R.\Delta\lambda}$$
$$= (1 - L)$$
$$= 0.0026.$$

This represents a ratio of distortion error to course length of about 3/10 of one per cent.

It is also found that if the point A remains stationary and the point B moves progressively around the latitude circle containing A, the ratio of error to course length will decrease progressively till a minimum value is reached when $\Delta\lambda = 180°$.

The Effect of Modification on the Scaled Azimuths

From Fig. 1 it is seen that on the true Lambert projection the azimuth at A of a course AB with both terminals on the same latitude circle is $90° - \frac{L.\Delta\lambda}{2}$, while the corresponding azimuth on the modified projection is $90° - \frac{\Delta\lambda}{2}$.

The error in azimuth at A, of the course AB, due to modification, therefore is $\frac{\Delta\lambda}{2}(1 - L)$ or $0.0013123\Delta\lambda$.

$$\begin{aligned} \text{For } \Delta\lambda &= 10°, \text{ error} = 1' \\ \Delta\lambda &= 90°, \quad " \quad = 7' \\ \Delta\lambda &= 180°, \quad " \quad = 14' \end{aligned}$$

As both the scale and azimuth errors resulting from modification are relatively small, they need not be considered a valid objection to the use of the modified projection for navigational or mapping purposes.

The Computation of the Map Elements

The formulae used in the computation of the various constants, the map elements, and the theoretical scale magnification are given below.

All computations are based on the Clarke Spheroid of 1866.

Semimajor axis a = 6 378 206.4 meters
Semiminor " b = 6 356 583.8 "

$$\text{Eccentricity} \quad e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\log e^2 = 7.8305026 - 10$$

$$\log \frac{b^2}{a^2} = 9.9970504 - 10$$

To Find Log tan $\frac{Z}{2}$, where Z is the isometric colatitude, we have

$$Z = 90 - \phi', \text{ and}$$

and

$$\text{Tan } \phi' = \frac{b^2}{a^2} \tan \phi$$

where ϕ' and ϕ are the geocentric and geodetic latitudes, respectively.
For latitude 71°,

$$\frac{Z}{2} = 9° 33' 36''.20$$

$$\log \tan \frac{Z}{2} = 9.22639448 - 10$$

Similarly for latitude 89° 59′ 58″,

$$\log \tan \frac{Z}{2} = 4.68860440 - 10$$

To Find The Constant A

$$A = \frac{(1 - e^2 \sin^2 \phi)^{\frac{1}{2}}}{a \sin 1''}.$$

Substituting and reducing, we get

$$\log A_{(\phi = 71°)} = 8.50840856 - 10$$
$$\log A_{(\phi = 89°59'58'')} = 8.50825180 - 10.$$

To Find The Constant L

$$L = \frac{\log \cos \phi_1 - \log \cos \phi_2 - \log A_1 + \log A_2}{\log \tan \frac{Z_1}{2} - \log \tan \frac{Z_2}{2}},$$

where ϕ_1, A_1 and Z_1 pertain to latitude 71° and ϕ_2, A_2 and Z_2 pertain to latitude 89° 59′ 58″. Inserting the numerical values of the above symbols and reducing, we get

$$L = 0.99737541$$
$$\log L = 9.99885866 - 10.$$

To Find The Constant K

$$K = \frac{\cos \phi_1}{A_1 \sin 1'' \, L \tan^L \frac{Z_1}{2}} = \frac{\cos \phi_2}{A_2 \sin 1'' \, L \tan^L \frac{Z_2}{2}}.$$

For latitude 71°, the southerly standard parallel, we have

$$\log A_1 = 8.5084085 - 10$$
$$\log \sin 1'' = 4.6855749 - 10$$
$$\log L = 9.9988587 - 10$$
$$\log \tan^L \frac{Z_1}{2} = 9.2284249 - 10$$
$$\overline{2.4212669 - 10}$$
$$\log \cos \phi_1 = 9.5126419 - 10$$
$$\overline{\log K = 7.0913750}$$

A similar computation for latitude 89° 59′ 58″, the latitude of the northerly standard parallel, gives the same value of log K.

With the constants L and K evaluated, the radii of the concentric latitude circles for each degree of latitude between 65° and the Pole are computed from the formula

$$R = K \tan^L \frac{Z}{2}.$$

The resulting values are given in Table I.

The Scale Error

For the modified projection, the scale error at any parallel of latitude will be the algebraic sum of the theoretical scale error of the true Lambert projection and the scale error due to modification. As the latter has already been shown to have little practical significance, the theoretical value may be considered applicable to the modified projection. Its value is derived from the formula

TABLE I

Latitude	R, meters	R, inches*	Percentage scale error	Latitude	R, meters	R, inches*	Percentage scale error
65	2,763,834	49.713	1.98	78	1,313,528	23.627	− 1.53
66	2,650,330	47.672	1.61	79	1,203,668	21.650	− 1.68
67	2,537,225	45.637	1.25	80	1,093,958	19.677	− 1.81
68	2,424,491	43.610	0.91	81	984,378	17.706	− 1.93
69	2,312,108	41.588	0.60	82	874,911	15.737	− 2.03
70	2,200,063	39.573	0.29	83	765,536	13.770	− 2.10
71	2,088,333	37.563	0.00	84	656,232	11.804	− 2.16
72	1,976,901	35.559	− 0.27	85	546,975	9.838	− 2.19
73	1,865,747	33.559	− 0.62	86	437,741	7.874	− 2.20
74	1,754,855	31.565	− 0.76	87	328,498	5.909	− 2.18
75	1,644,205	29.574	− 0.98	88	219,205	3.943	− 2.11
76	1,533,778	27.588	− 1.18	89	109,794	1.975	− 1.96
77	1,423,562	25.606	− 1.36	90	0	0.000	0.00

* *Applicable to map on scale of 30 nautical miles to the inch.*

$$m = \frac{LRA \sin 1''}{\cos \phi},$$

where m is the magnification of the scale along any parallel of latitude. The percentage ratio of scale error to length of course therefore is $100\ (1 - m)$.

For latitude 71°,

$$\begin{aligned} \log L &= 9.998\ 858\ 66 - 10 \\ \text{“ } R &= 6.319\ 799\ 82 \\ \text{“ } A &= 8.508\ 408\ 56 - 10 \\ \log \sin 1'' &= \underline{4.685\ 574\ 87 - 10} \\ &\quad 9.512\ 641\ 91 - 10 \\ \log \cos 71° &= 9.512\ 641\ 91 - 10 \\ \text{“ } m &= 0 \\ m &= 1 \\ \text{Scale error} &= 0 \end{aligned}$$

At latitude 71°, the location of the southerly standard parallel, the scale error should of course be equal to zero.

For latitude 84°,

$$\begin{aligned} \log L &= 9.998\ 858\ 66 - 10 \\ \text{“ } R &= 5.817\ 057\ 28 \\ \text{“ } A &= 8.508\ 267\ 94 - 10 \\ \log \sin 1'' &= \underline{4.685\ 574\ 87 - 10} \\ &\quad 9.009\ 758\ 75 - 10 \\ \log \cos 84° &= \underline{9.019\ 234\ 57 - 10} \\ \text{“ } m &= 9.990\ 524 \quad - 10 \\ m &= 0.9784 \\ \text{Scale error} &= 2.16\%. \end{aligned}$$

Table I gives the radii of the latitude circles in meters and in inches for a map scale of 30 nautical miles to the inch. In addition, the theoretical scale errors on a true Lambert projection are given in column three for each degree of latitude between 65° and the North Pole.

Table II gives the values of log A.

TABLE II

Latitude	Log A, meters	Latitude	Log A, meters	Latitude	Log A, meters
65	8.5085159 − 10	74	8.5083643	83	8.5082738
66	8.5084964	75	8.5083509	84	8.5082680
67	8.5084776	76	8.5083384	85	8.5082631
68	8.5084594	77	8.5083267	86	8.5082590
69	8.5084418	78	8.5083158	87	8.5082559
70	8.5084248	79	8.5083056	88	8.5082536
71	8.5084086	80	8.5082964	89	8.5082523 − 10
72	8.5083930	81	8.5082880		
73	8.5083783	82	8.5082805		

From the tabulation, it is seen that the error at the outer edge of the map is +1.98%, at latitude 71° and at the North Pole it is zero, and at latitude 86° it is − 2.20%. Owing to the compensating effect of the positive and negative signs of the errors at different parts of the map, the over-all scaled value of a long course is usually correct to within 1%.

Examples of Great-Circle Courses Plotted on the Modified Lambert Projection

To demonstrate the accuracy with which azimuths and distances may be scaled from the projection, a number of long geodetic lines* joining assumed terminal points on the earth's surface were plotted on a Modified Lambert Projection. To facilitate the demonstration, the latitudes and longitudes of a number of intermediate points on each course were computed by precise geodetic formulae. The initial azimuths and the azimuths† of the courses at each intermediate station, and the distances between intermediate points, were also computed so that the scaled values could be accurately compared.

Table III gives the latitudes and longitudes of the intermediate points on each course. It also gives the theoretical and scaled azimuths of the geodetic line at each point on the course and the differences between the theoretical and scaled values. It will be noted that, for the chosen lines, the maximum error in the scaled azimuth at any point was about one-third of a degree.

** A geodetic line is the shortest line joining two points on the earth's surface treated as an ellipsoid of revolution.*

Aerial navigators usually consider their flight lines as great-circle courses on a sphere having a radius equal to the mean radius of the earth. Azimuths and lengths of the courses computed on this assumption, while not strictly correct, are usually well within the limits of accuracy required for aerial navigation.

† In Canadian survey practice, azimuths are reckoned clockwise from the north from 0° to 360°.

TABLE III

COMPARISON OF AZIMUTHS (GREAT CIRCLE COURSE)

Point	Latitude	Longitude	Azimuth		Difference
			Computed	Scaled	
Course A	*From Point 1* {*Lat. 65° N.* *Long. 145°59′ 42″.4 W.* *to Point 7* {*Lat. 80° N.* *Long. 10°21′ 36″.4 E.* *Length = 2071.1 nautical miles*				
1	65° N.	145° 59′ 42″.4 W.	7° 05′ 13″.6	7° 15′	10′
2	68° N.	145° 00′ 00″.0 W.	8° 00′ 00″.0	7° 55′	5′
3	74° N.	141° 56′ 12″.2 W.	10° 54′ 01″.5	10° 40′	14′
4	80° N.	135° 12′ 32″.9 W.	17° 27′ 53″.9	17° 30′	2′
5	87° N.	67° 44′ 57″.0 W.	84° 40′ 57″.4	84° 30′	11′
6	87° N.	57° 05′ 59″.5 W.	95° 19′ 02″.6	95° 15′	4′
7	80° N.	10° 21′ 36″.4 E.	162° 32′ 06″.3	162° 30′	2′
Course B	*From Point 1* {*Lat. 65° N.* *Long. 129°01′ 12″.6 W.* *to Point 5* {*Lat. 79° N.* *Long. 51°51′ 18″.4 W.* *Length = 1495.7 nautical miles*				
1	65° N.	129° 01′ 12″.6 W.	26° 18′ 42″.3	26° 30′	11′
2	68° N.	125° 00′ 00″.0 W.	30° 00′ 00″.0	29° 45′	15′
3	74° N.	111° 29′ 33″.4 W.	42° 47′ 42″.5	42° 45′	3′
4	79° N.	74° 28′ 04″.0 W.	78° 53′ 46″.3	79° 10′	16′
5	79° N.	51° 51′ 18″.4 W.	101° 06′ 13″.7	100° 55′	11′
Course C	*From Point 1* {*Lat. 65° N.* *Long. 100° W.* *to Point 4* {*Lat. 73° N.* *Long. 61° 27′ 21″.6 W.* *Length = 937.2 nautical miles*				
1	65° N.	100° 00′ 00″.0 W.	42° 47′ 47″.0	42° 40′	8′
2	68° N.	92° 07′ 04″.4 W.	50° 01′ 44″.6	50° 00′	2′
3	71° N.	79° 30′ 36″.2 W.	61° 50′ 54″.0	61° 50′	1′
4	73° N.	61° 27′ 21″.6 W.	79° 02′ 05″.3	78° 50′	12′
Course D	*From Point 1* {*Lat. 65° N.* *Long. 96°04′ 20″ W.* *to Point 3* {*Lat. 70° N.* *Long. 71°49′ 23″.8 W.* *Length = 628.8 nautical miles*				
1	65° N.	96° 04′ 20″.0 W.	50° 50′ 15″.4	50° 30′	20′
2	68° N.	85° 00′ 00″.0 W.	61° 00′ 00″.0	61° 10′	10′
3	70° N.	71° 49′ 23″.8 W.	73° 18′ 37″.2	73° 35′	16′
Course E	*From Point 1* {*Lat. 65° N.* *Long. 88° 16′ 40″ W.* *to Point 3* {*Lat. 68° N.* *Long. 65° 00′ 00″.0 W.* *Length = 583.4 nautical miles*				
1	65° N.	88° 16′ 40″.0 W.	61° 38′ 23″.1	61° 25′	13′
2	67° N.	76° 48′ 44″.4 W.	72° 07′ 16″.7	72° 10′	3′
3	68° N.	65° 00′ 00″.0 W.	83° 02′ 26″.0	83° 15′	13′

TABLE III (continued)

COMPARISON OF AZIMUTHS (GREAT CIRCLE COURSE)

Point	Latitude	Longitude	Azimuth		Difference
			Computed	Scaled	
Course F	*From Point 1* {*Lat. 65° N.* *Long. 120° W.*		*to Point 9* {*Lat. 75° 15′ N.* *Long. 45° 51′ 25″.0 W.*		
		Length = 1509.8 nautical miles			
1	65°	120° 00′ 00″.0 W.	35° 20′ 00″.0	35° 15′	5′
2	67°	116° 17′ 48″.8 W.	38° 43′ 01″.2	38° 25′	18′
3	69°	111° 41′ 12″.0 W.	42° 59′ 33″.0	43° 00′	0′
4	71°	105° 40′ 51″.0 W.	48° 38′ 15″.3	48° 40′	2′
5	73°	97° 13′ 03″.0 W.	56° 41′ 21″.5	56° 40′	1′
6	74° 30′	87° 25′ 02″.4 W.	66° 06′ 01″.9	66° 15′	9′
7	75° 30′	75° 45′ 27″.6 W.	77° 21′ 57″.2	77° 10′	12′
8	75° 51′ 31″.1	62° 43′ 18″.8 W.	90° 00′ 00″.0	90° 10′	10′
9	75° 15′	45° 51′ 25″.0 W.	106° 20′ 21″.5	106° 35′	15′

Table IV shows the theoretical and scaled distances, respectively, between the intermediate points on the lines. It also shows the difference between the two values expressed as a percentage of the scaled distance. It is seen that the maximum ratio error in a scaled distance is 2.2%.

Long great-circle courses will in general be slightly curved lines concave towards the pole. They diverge from the map chord, which is the straight line joining the terminal points of any course. Fig. 3 shows the divergence between the map chords and the great-circle courses for each of the six experimental flight lines *A*, *B*, *C*, *D*, *E* and *F*.

Nomogram for Plotting Great-Circle Courses

For laying down great-circle courses graphically on the projection with speed and accuracy, a nomogram shown in Fig. 2 has been developed. It gives at once the offset distance at the center of the map chord to the great-circle course and the angle in degrees at each terminal point between the map chord and the curved line constituting the great-circle course.

Points may also be established by offset from the chord at quarter-length intervals. This, however, should not be necessary except for extremely long courses.

The nomogram is applicable only to the full size map of Arctic Canada on a scale of 30 nautical miles to the inch. To be applicable to the linecut reproduction of the map shown in Fig. 3 (about one-fourteenth original scale), the numerical values of the chord length in inches and the perpendicular distance in inches on the nomogram scale shown in Fig. 2 would have to be similarly reduced.

TABLE IV

COMPARISON OF DISTANCES (NAUTICAL MILES)

Points	Distance		Difference	Percentage error
	Computed	Scaled		
Course A				
1–2	182.1	184.2	+ 2.1	+ 1.1
2–3	366.1	367.8	+ 1.7	+ 0.5
3–4	372.3	365.7	− 6.6	− 1.8
4–5	558.7	546.0	− 12.7	− 2.2
5–6	33.2	32.7	− 0.5	− 1.5
6–7	558.7	545.9	− 12.8	− 2.2
1–7	2071.1	2040.9*	− 30.2	− 1.5
Course B				
1–2	204.6	207.3	+2.7	+ 1.3
2–3	445.8	447.0	+1.2	+ 0.3
3–4	586.9	578.1	− 8.8	− 1.5
4–5	258.4	255.0	− 3.4	− 1.3
1–5	1495.7	1486.8*	− 8.9	− 0.6
Course C				
1–2	261.2	266.1	+ 4.9	+ 1.9
2–3	320.5	322.2	+ 1.7	+ 0.5
3–4	355.5	354.0	− 1.5	− 0.4
1–4	937.2	942.0*	+ 4.8	+ 0.5
Course D				
1–2	320.7	325.3	+ 4.6	+ 1.4
2–3	308.1	310.8	+ 2.7	+ 0.9
1–3	628.8	636.1*	+ 7.3	+ 1.1
Course E				
1–2	305.0	309.3	+ 4.3	+ 1.4
2–3	278.4	280.8	+ 2.4	+ 0.9
1–3	583.4	590.1*	+ 6.7	+ 1.2
Course F				
1–2	150.7	153.3	+ 2.6	+ 1.7
2–3	159.0	160.5	+ 1.5	+ 0.9
3–4	172.5	173.1	+ 0.6	+ 0.3
4–5	198.0	198.3	+ 0.3	+ 0.2
5–6	188.0	186.9	− 1.1	− 0.6
6–7	191.2	189.3	− 1.9	− 1.0
7–8	195.1	193.2	− 1.9	− 1.0
8–9	255.3	255.6	+ 0.3	+ 0.1
1–9	1509.8	1506.6*	+ 3.2	+ 0.2

* *The scaled distance along the straight line joining the terminals is slightly less than the sum of the component parts of the great-circle course.*

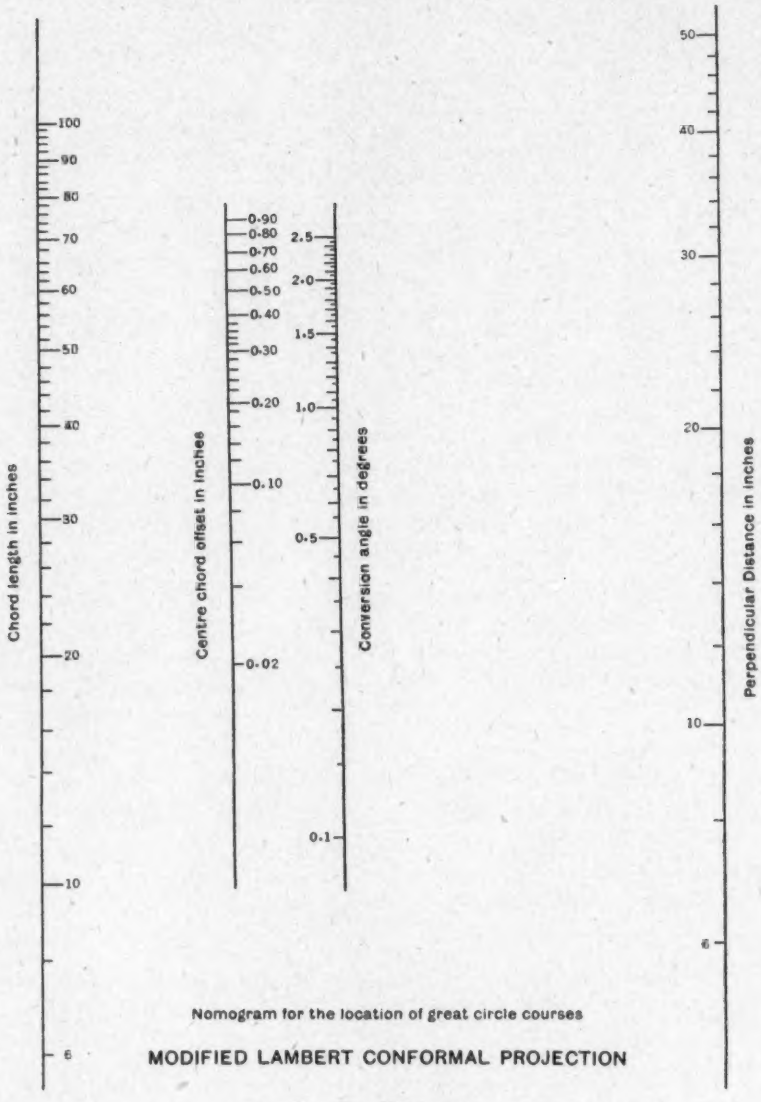

FIG. 2. *Nomogram.*

Directions for Using the Nomogram

(1) On the map, join the plotted terminal points of the course to be flown with a straight line to form the map chord.

(2) Measure the length of the map chord in inches.

(3) Measure the perpendicular distance from the geographical pole to the chord.

(4) Place a straight edge on the nomogram with one end on left-hand scale at the reading corresponding to the chord length and the other end on the right-hand scale at the reading corresponding to the perpendicular distance.

(5) At the point where the straight edge intersects the center-chord offset scale, read off the value of the offset distance from the center of the chord to the great-circle course. The latter will always be on the side of the chord remote from the pole.

(6) If more points are required on the great-circle course, join the newly established central point with each terminal, forming two new chords. The offset distance to the great-circle course from the center of each new chord will be one-quarter of the original offset distance.

(7) The initial azimuth of the great-circle course may be readily derived by adding (or subtracting) the value of the conversion angle between the chord and the great circle course. This angle may be taken directly from the nomogram by setting a straight edge at the corresponding reading of the chord length and perpendicular distance on the left and right-hand scale, respectively, and noting where the straight edge intersects the conversion angle scale. The conversion angle is added to the scaled azimuth of the chord for all azimuths between 0° and 180° and subtracted for all azimuths between 180° and 360°.

By laying off the conversion angles on the chart from the terminals of the chord, and establishing the central point on the great-circle course by offset distance from the map chord, the great-circle course may be drawn with sufficient accuracy for ordinary navigational purposes.

Numerical Example—Course F

Scaled chord length	= 50″.3
Scaled perpendicular distance	= 27″.6
From the nomogram, we get	
(*a*) Center chord offset	= 0″.36
(*b*) Conversion angles at points 1 and 9 between chord and great-circle course	≐ 1° 30′

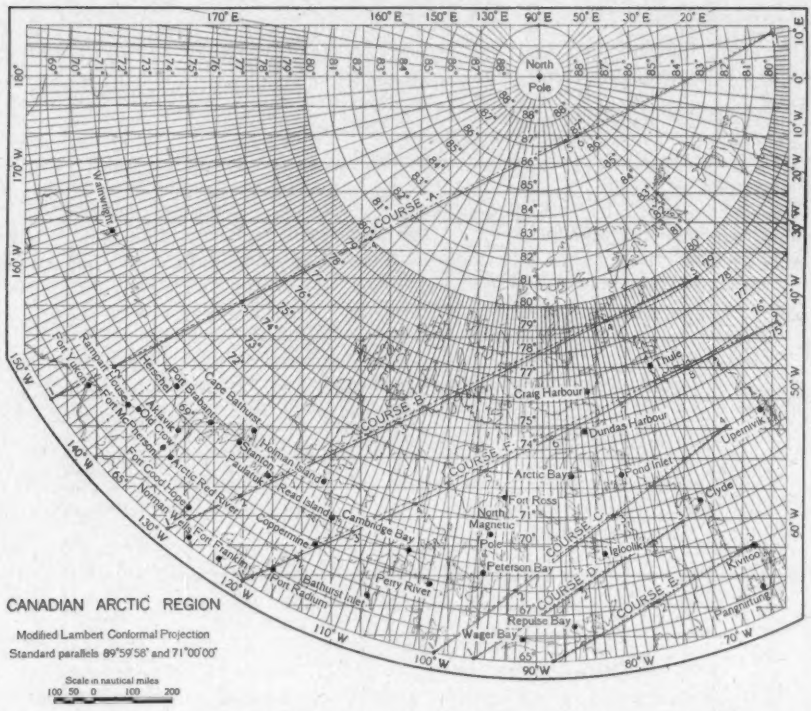

FIG. 3. *Reproduction of Map of Arctic Canada—on Modified Lambert conformal projection.*

Adding to (*b*) the scaled value of the initial azimuth of map chord (33° 45′), we get 35° 15′ as the initial azimuth of the great-circle course. The calculated initial azimuth is given in Table III as 35° 20′, differing only by 5′ from the value derived by the graphical method.

The value of the center chord offset for the map chord to the great-circle course scaled directly from the plotted position of Course *F* (by Table III) is 0″.37, giving an agreement to 1/100 in. with the value derived from the nomogram.

With the aid of a flexible ruler or baton, the two terminal points and the center point of the great-circle course may be joined by a curved line giving an almost exact representation of the true great-circle course.

General Description of the Map

Fig. 3 shows a reproduction of the finished map, upon which the six great-circle courses have been plotted. The spacing and numbering* of meridians and latitude circles, the faint delineation of land masses and topographical

**On the small-scale reproduction of the map shown in Fig. 3, it was not possible to show all the numbering of meridians and parallels of latitude.*

features, and the marking of settlements and air bases are, in nearly all cases, in conformity with suggestions put forward by officers of the United States Army Air Force and the Royal Canadian Air Force. The square grid system with lines parallel to and at right angles to the Greenwich Meridian was included to facilitate navigation by the "*G*" system first proposed by W/C K. C. Maclure of the R.C.A.F., who flew over the North Pole in the Aries in 1945.

Airmen who have made extensive use of this map on flights over polar and subpolar areas have reported the following desirable features:—

(*a*) The limits of the projection extend from latitude 65° N. to 2° beyond the pole.

(*b*) With the standard parallels at latitude 89° 59′ 58″ and 71° 00′ 00″, the scale errors over the map as a whole are practically negligible.

(*c*) The azimuths or true bearings of great-circle courses may be scaled with sufficient accuracy from the chart.

(*d*) The scale of 30 nautical miles to the inch is satisfactory.

(*e*) The omission of isogonals until such time as more reliable magnetic data are available.

(*f*) The prominence of latitude and longitude designations.

(*g*) The decreasing number of meridians in the polar region lessens congestion.

(*h*) The grid lines are conveniently spaced.

The navigational map of Arctic Canada has, however, one undesirable feature common to all maps of our subpolar areas. As the systematic mapping of areas north of latitude 65° has been inaugurated in recent years only, there are insufficient survey data available at the present time to make possible the correct delineation of the polar land masses and details of topography. At the completion of several years of the present mapping program, this condition should no longer obtain.